Combinatorics

Mid-Term Examination

Instructions: All questions carry equal marks.

1. Show that a binary code of length 6 and minimum distance 3 can have at most 8 codewords.

2. Define self-dual codes. Give an example each of binary self-dual code of length 4,6 and 8.

3. Let G be a graph on v vertices, each of degree at most k. Assume that any two adjacent vertices have at least λ common neighbours and any two non-adjacent vertices have at least μ common neighbours. Then show that
$$k(k-1-\lambda) \geq \mu((v-k-1).$$
Prove that equality holds if and only if G is a $srg(v,k,\lambda,\mu)$ graph.

4. Define a $t-(v,k,\lambda)$ design. Prove that there exists a $2-(15,3,1)$ design.

5. Prove that in any non-trivial $t-(v,k,1)$ design, we must have
$$v \geq (t+1)(k+1-t).$$